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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, JUNE 2022

FIRST YEAR [BATCH 2021-24]

Date : 20/06/2022 **PHYSICS (HONOURS)**

Time : 11 am – 1 pm **Paper : III [CC3]** Full Marks : 50

Group :A

Answer **any five** questions: [5×10]

1. a) Write down the complex form of Fourier series . Obtain the complex form of the Fourier series of the function $f(x) = |x| \quad -\pi \le x \le \pi$

$$f(x+2\pi) = f(x)$$

b) Find the Fourier transform of the Gaussian function $f(x) = e^{-\alpha x^2} \quad (\alpha > 0, constant)$ [(1+5)+4]

2. a) Test for convergence of the series $\sum_{n=1}^{\propto} \frac{n!3^n}{n^n}$

b) Can you expand $f(x) = \tan x$ in Fourier series? Explain.

c) Let F(x) have a Fourier series expansion

$$F(x) = \sum_{n=1}^{\propto} a_n \cos nx + \sum_{n=1}^{\propto} b_n \sin nx$$

then prove that $< F^2(x) >= \frac{1}{2\pi}\int_{-\pi}^{\pi} F^2(x)dx = \sum_{n=1}^{\propto}(a_n^2 + b_n^2)/2$

d) The function $f(x) = x^2$ is defined within the interval $-\pi \le x \le \pi$ and outside it is periodic.

Expand f(x) in a Fourier series to show

$$f(x) = \frac{\pi^2}{3}\sum_{n=1}^{\propto}\frac{4}{n^2}(-1)^n \cos nx$$ [2+1+2+5]

3. a) Find out the general solution of the following partial differential equation by the method of separation of variables

$$\frac{\partial^2 T}{\partial x^2} = \alpha^2 \frac{\partial T}{\partial t}$$

b) Determine the condition under which the following differential equation can be solved using the method of separation of variables

$$c_1 \frac{\partial \phi}{\partial t} + c_2 \nabla^2 \phi + v(x,t)\phi(x,t) = 0 \text{ where } c_1 \ and\ c_2\ are \text{ constants}$$ [6+4]

4. a) Solve the following equation using the method of separation of variables

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

with V=0 when y = 0
V=0 when y = π
$V = V_0$ when x=1
V(-x,y)=V(x,y)

b) $+\phi = \phi(x, y, z)$ write $\nabla^2\phi = 0$ in spherical polar co-ordinate system. Assuming separation of variable show that Laplace's equation can be decoupled into three total differential equations. [6+4]

5. Evaluate the following integral using β and γ functions [5×2]

a) $\int_0^\infty \frac{x^8(a-x^6)}{(1+x)^{24}} dx$

b) $\int_{\_}^{1} (ln \frac{1}{\zeta})^{n-1} d\zeta$

6. Prove the followings [5×2]

a) $\beta(\theta, \phi) = \frac{\Gamma(\theta)\Gamma(\phi)}{\Gamma(\theta+\phi)}$

b) $\Gamma \frac{1}{2} = \sqrt{\pi}$

7. Find the solution of the following equation [10]

$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$

8. Study the following distribution function

x	0	1	2	3	4	5	6	7
$\wp(x)$	0	φ	$2\,\varphi$	$2\,\varphi$	$3\,\varphi$	φ^2	$2\,\varphi^2$	$7\,\varphi^2 + \varphi$

Find

a) φ =?

b) $\wp(x < 6)$, $\wp(x \geq 6)$, $\wp(0 < x < 5)$

c) Distribution Function

d) If $\wp(x \leq \epsilon) > \frac{1}{2}$, then find minimum value of ϵ.

e) Find $\wp(\frac{1.5<x<4.5}{x>2})$ [1+3+2+2+2]

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